Tutorial Problem sheet 6:

3D Dynamics

DE2 EA2.1 SOLID MECHNICS 2

Q1-8: Euler's equations
Q9-11: Parallel axis theorem

REF: Engineering Mechanics: Statics and Dynamics. Fourth Edition. Bedford and Fowler. Chapter 20.

Comments and corrections to connor.myant@imperial.ac.uk
Lecture resources may be found on Blackboard

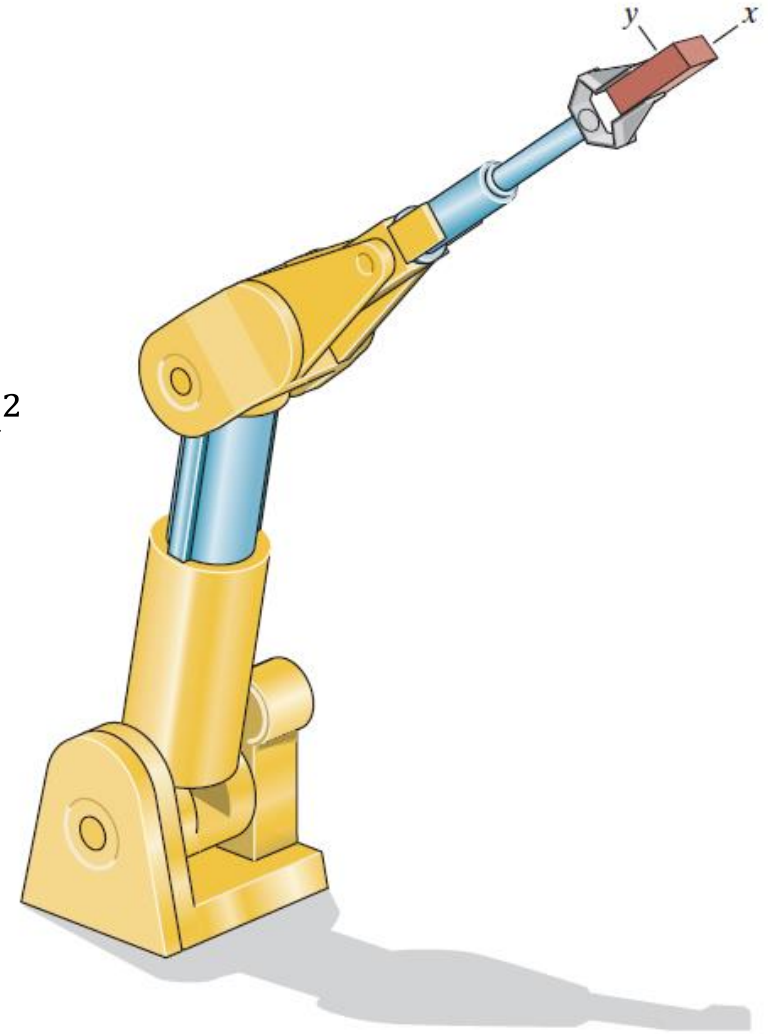
Q1

A robotic manipulator moves a casting. The inertia matrix of the casting in terms of a body-fixed coordinate system with its origin at the center of mass is;

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & -I_{yy} & -I_{yz} \\ -Izx & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.05 & -0.03 & 0 \\ -0.03 & 0.08 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \text{kg-}m^2$$

At the present instant, the angular velocity and angular acceleration vectors of the casting are $\vec{\omega} = 1.2i + 0.8j - 0.4k$ (rad/s) and $\vec{\alpha} = 0.26i - 0.07j + 0.13k$ (rad/s^2).

What moment is exerted about the center of mass of the casting by the manipulator?

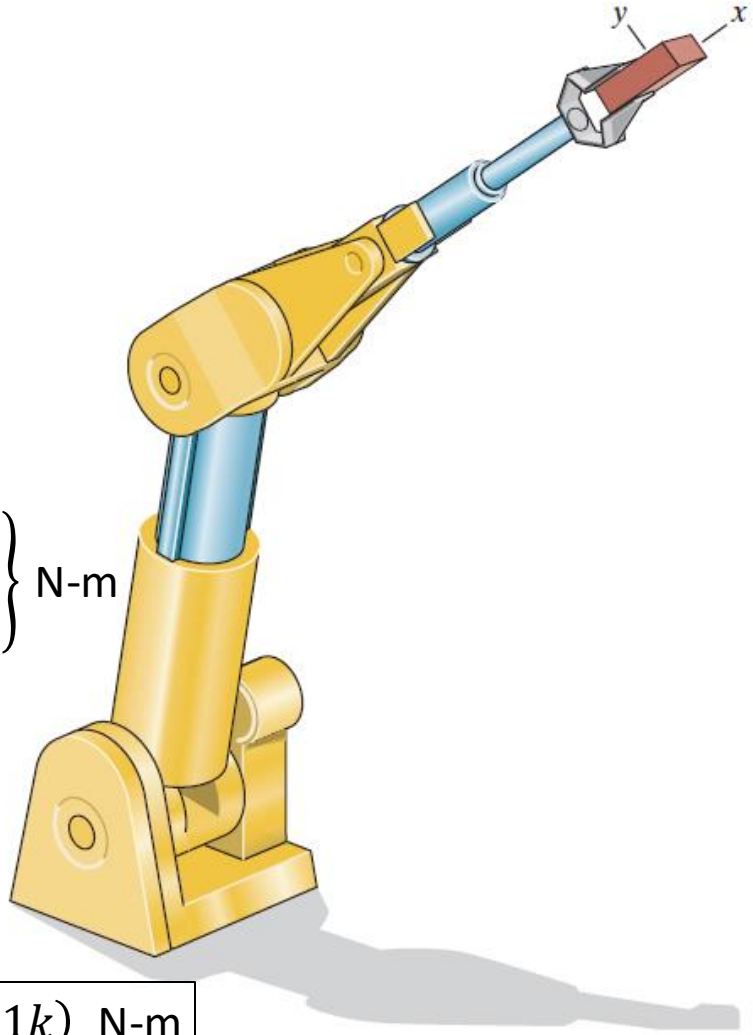


Q1

Solution:

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} 0.05 & -0.03 & 0 \\ -0.03 & 0.08 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \begin{Bmatrix} 0.26 \\ -0.07 \\ 0.13 \end{Bmatrix} \text{N-m}$$

$$+ \begin{bmatrix} 0 & 0.4 & 0.8 \\ -0.4 & 0 & -1.2 \\ -0.8 & 1.2 & 0 \end{bmatrix} \begin{bmatrix} 0.05 & -0.03 & 0 \\ -0.03 & 0.08 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \times \begin{Bmatrix} 1.2 \\ 0.8 \\ -0.4 \end{Bmatrix} \text{N-m}$$



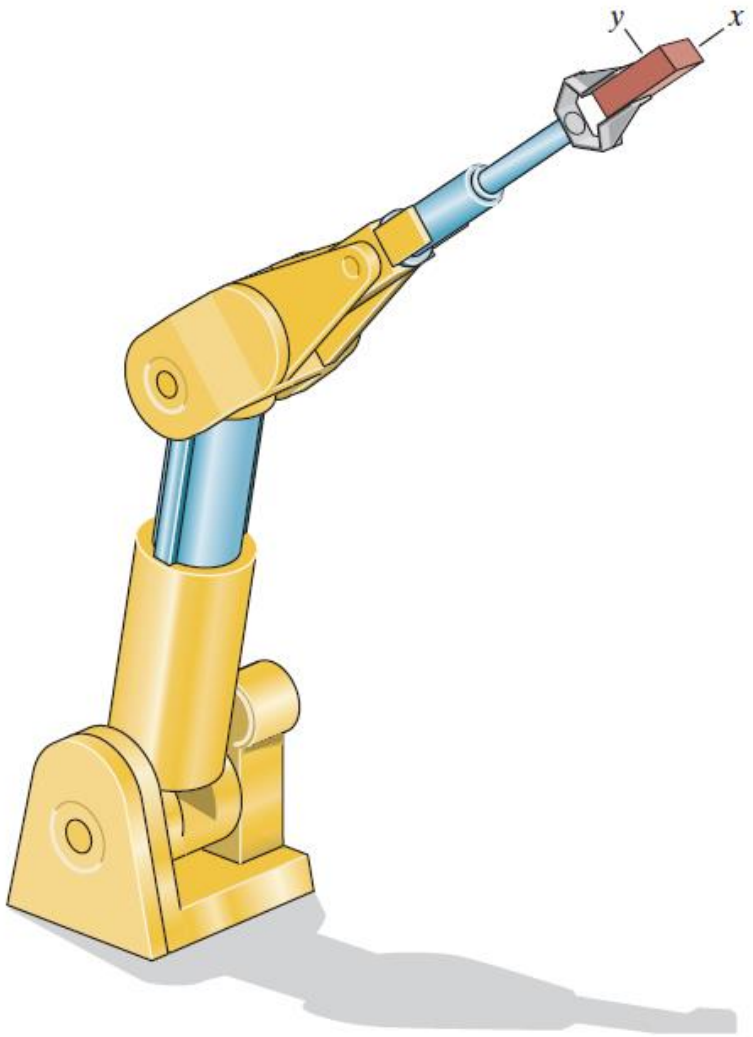
$$\overrightarrow{\mathrm{M}} = \left(M_x i + M_y j + M_z k\right) = (0.0135i - 0.0086j + 0.01k) \text{ N-m}$$

Q2

A robotic manipulator holds a casting. The inertia matrix of the casting in terms of a body-fixed coordinate system with its origin at the center of mass is;

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & -I_{yy} & -I_{yz} \\ -Izx & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.05 & -0.03 & 0 \\ -0.03 & 0.08 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \text{kg-}m^2$$

At the present instant, the casting is stationary. If the manipulator exerts a moment $\Sigma\vec{M} = 0.042i + 0.036j + 0.066k$ (N m) about the center of mass, what is the angular acceleration of the casting at that instant?



Strategy: $\Sigma M = I\alpha$

$\vec{\alpha} = (\alpha_x i + \alpha_y j + \alpha_z k)$=(1.43i + 0.987j + 1.65k) rad/s^2

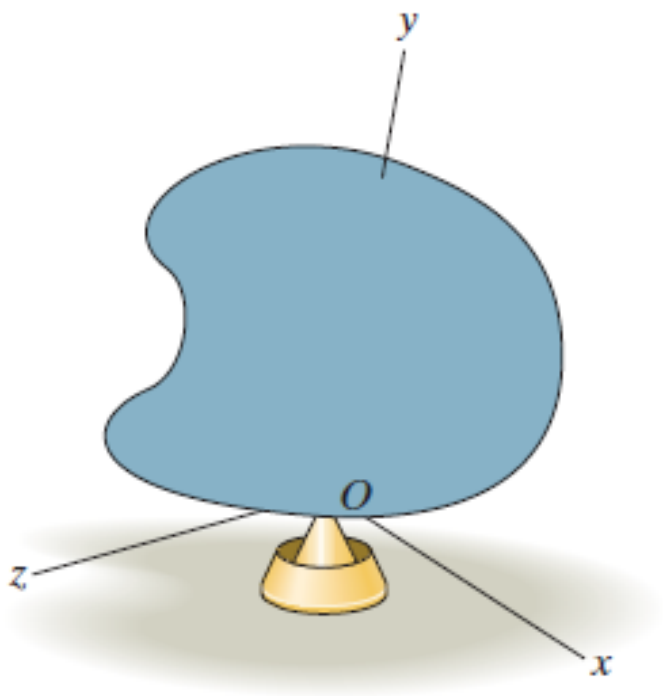
Q3

The rigid body rotates about the fixed point O. Its inertia matrix in terms of the body-fixed coordinate system is;

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & -I_{yy} & -I_{yz} \\ -Izx & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 5 \end{bmatrix} \text{ kg-}m^2$$

At the present instant, the rigid body's angular velocity vector is $\vec{\omega} = 6i + 6j - 4k$ (rad/s) and its angular acceleration is zero.

What total moment about O is being exerted on the rigid body?



Strategy: Consider equation 1, but remember that the angular acceleration is zero.

$$\vec{M} = \left(M_x i + M_y j + M_z k\right) = (-76i + 36j - 60k) \text{ N-m}$$

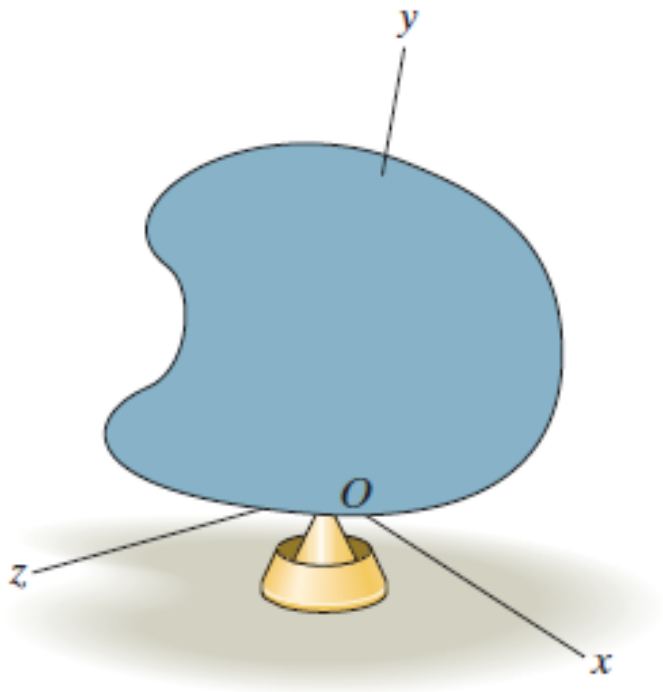
Q4

The rigid body rotates about the fixed point O. Its inertia matrix in terms of the body-fixed coordinate system is;

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & -I_{yy} & -I_{yz} \\ -Izx & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 5 \end{bmatrix} \text{ kg-}m^2$$

At the present instant, the rigid body's angular velocity is $\vec{\omega} = 6i + 6j - 4k$ (rad/s). The total moment about O due to the forces and couples acting on the rigid body is zero.

What is its angular acceleration?

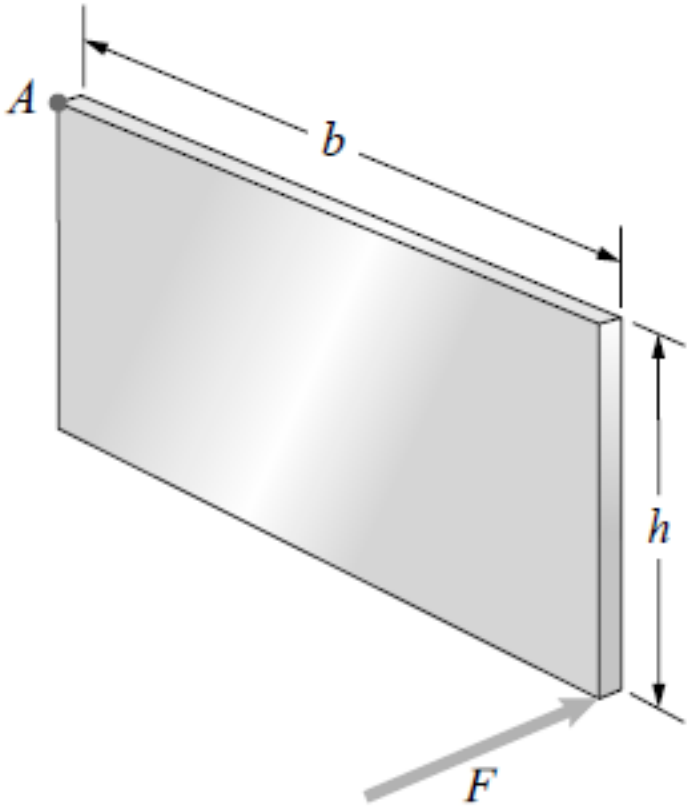


$\vec{\alpha}$ = (16.2i - 5.56j + 13.1k) rad/s^2

Q5

The dimensions of the 20 kg thin plate are $h = 0.4$ m and $b = 0.6$ m. The plate is stationary relative to an inertial reference frame when the force $F = 10$ N is applied in the direction perpendicular to the plate. No other forces or couples act on the plate.

At the instant F is applied, what is the magnitude of the acceleration of point A relative to the inertial reference frame?

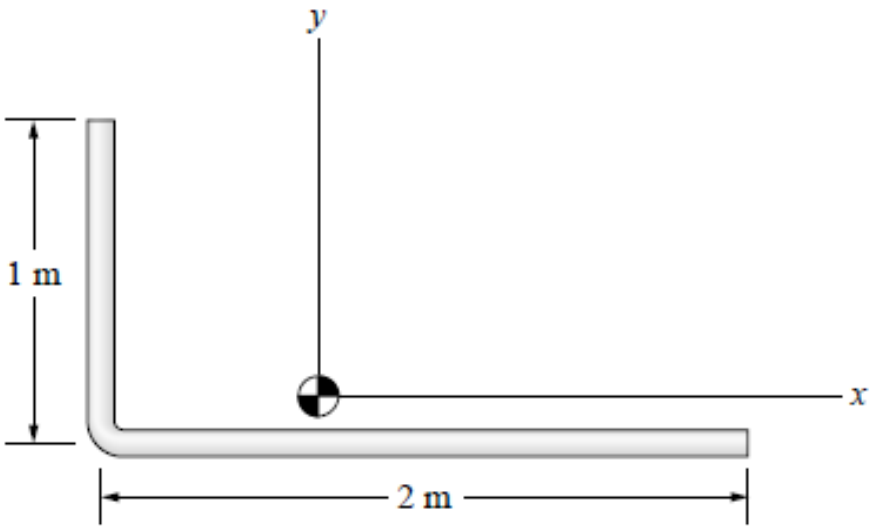


$|a_A| = 2.5\ m/s^2$

Q6

In terms of the coordinate system shown, the inertia matrix of the 6-kg slender bar is:

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & -I_{yy} & -I_{yz} \\ -Izx & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.667 & 0 \\ 0.667 & 2.667 & 0 \\ 0 & 0 & 3.167 \end{bmatrix} \text{kg-}m^2$$



The bar is stationary relative to an inertial reference frame when the force F = 12k (N) is applied at the right end of the bar. No other forces or couples act on the bar. Determine;

(a) the bar's angular acceleration vector relative to the inertial reference frame and
(b) the acceleration of the right end of the bar relative to the inertial reference frame at the instant the force is applied.

Strategy: a) First consider the coordinates for the centre of mass. Now take moments around this point and consider Eq. (2). b) Think about the kinematic relation ship between points.

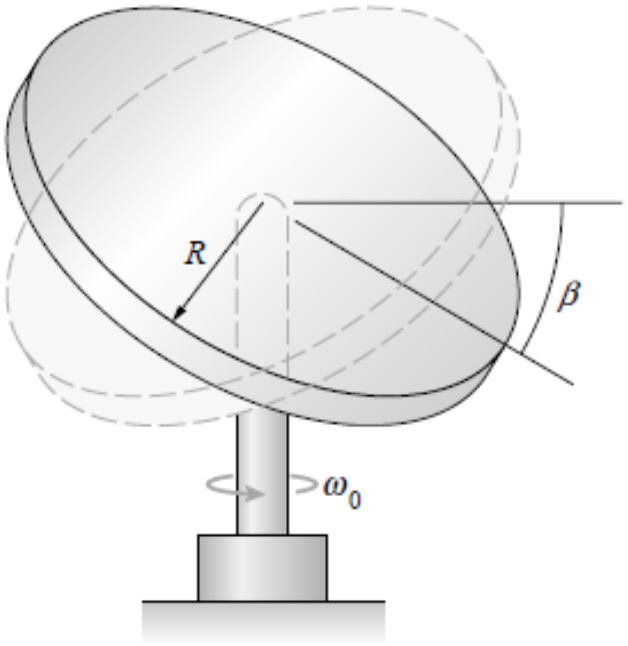
a) $\vec{\alpha} = 6.01\boldsymbol{i} - 7.50\boldsymbol{j}\ \left(\frac{rad}{s^2}\right)$

b) $\vec{a}_A = 11k\ (\frac{m}{s^2})$

Q7

The thin circular disk of radius R = 0.2 m and mass m = 4 kg is rigidly attached to the vertical shaft. The plane of the disk is slanted at an angle $\beta = 30^\circ$ relative to the horizontal. The shaft rotates with constant angular velocity $\omega_0 = 25$ rad/s.

Determine the magnitude of the couple exerted on the disk by the shaft.



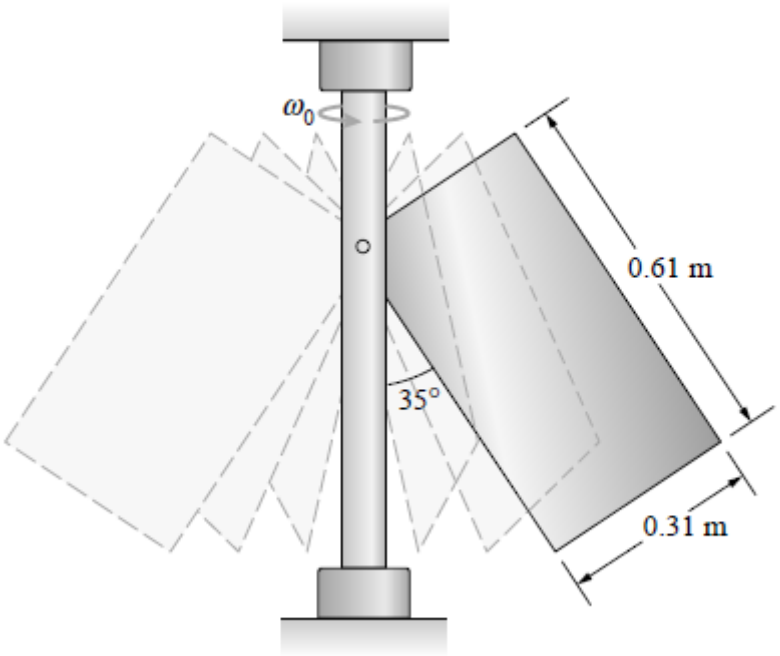
Strategy: Make sure you draw a free body diagram of the disk, and consider its body fixed reference frame. You can then build its inertia matrix using the formula sheet. Once you have this plug into Eq. (2).

$$\Sigma\vec{M} = 10.8i\ N\ m$$

Q8

The vertical shaft rotates with constant angular velocity ω_0. The 35° angle between the edge of the 44.5 N thin rectangular plate pinned to the shaft and the shaft remains constant.

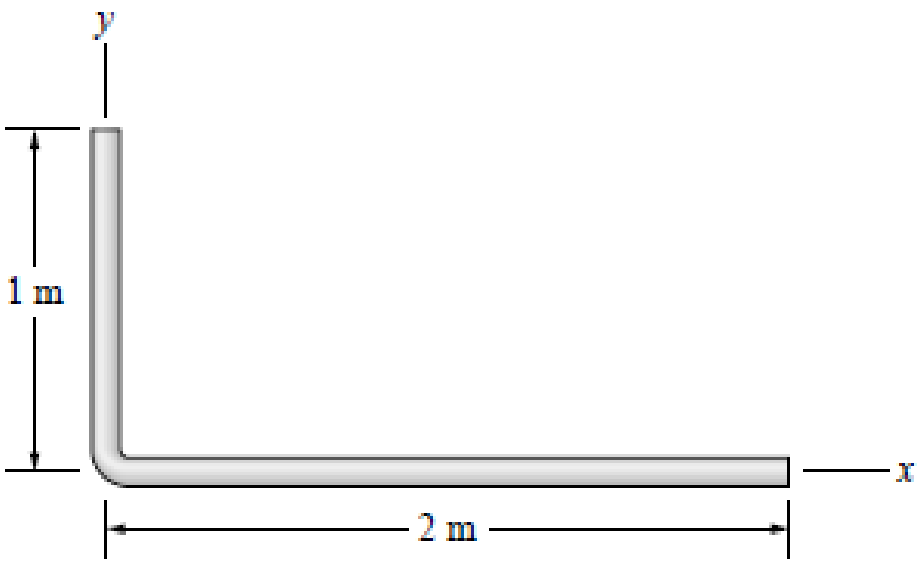
Determine ω_0.



$\omega_0 = 7.025$ rad/s

Q9

The mass of the bar is 6 kg. Determine the moments and products of inertia of the bar in terms of the coordinate system shown.



Q9 Parallel Axis theorem

The moment of inertia in the coordinate system shown can be derived by inspection by taking the moment of inertia of each element about the origin: from the formula sheet the moments of inertia about the origin of the slender bars are;

$$I_{xx} = \frac{1}{3} m_v l_v^2$$
$$I_{yy} = \frac{1}{3} m_h l_h^2$$

and $I_{zz} = I_{xx} + I_{yy}$, where the subscripts v and h denote the vertical and horizontal bars respectively. Noting that the masses are

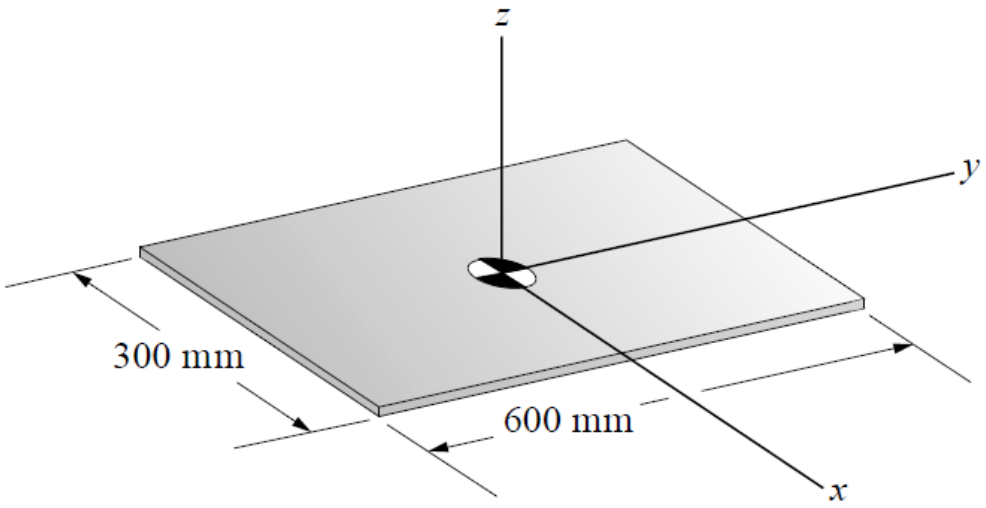
$$m_v = \frac{m l_v}{l_v + l_h}$$
$$m_h = \frac{m l_h}{l_v + l_h}$$

the moment of inertia matrix becomes:

$$[I] = \begin{bmatrix} 0.6667 & 0 & 0 \\ 0 & 5.333 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{kg-m}^2.$$

Q10

The 4-kg thin rectangular plate lies in the x-y plane. Determine the moments and products of inertia of the plate in terms of the coordinate system shown.



Strategy: Use the formula sheet and find the moments and products of inertia for a thin plate.

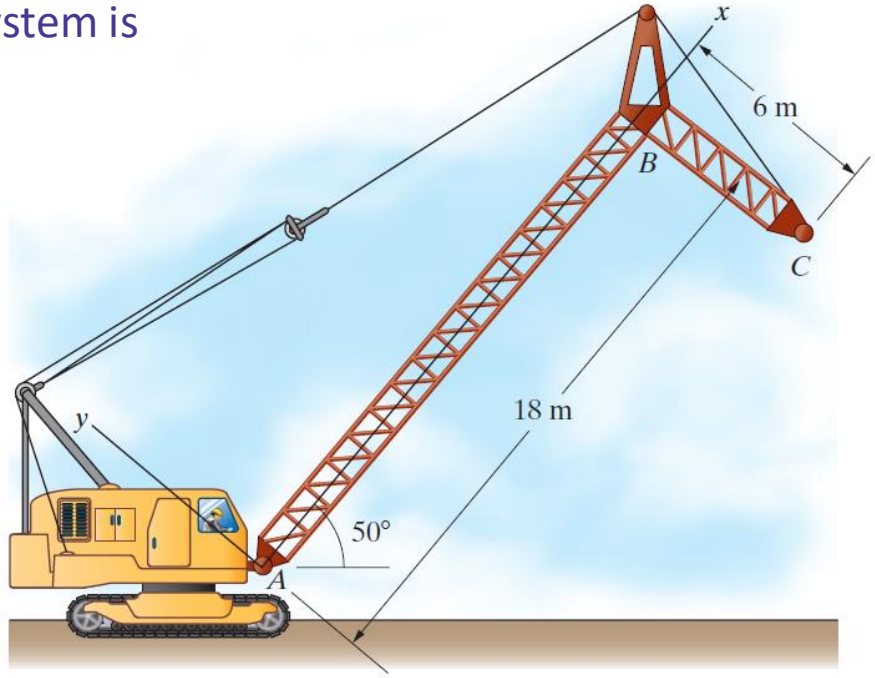
$$[I] = \begin{bmatrix} 0.12 & 0 & 0 \\ 0 & 0.03 & 0 \\ 0 & 0 & 0.15 \end{bmatrix} \text{ kg-m}^2.$$

Q11

Determine the moments and products of inertia of the object in terms of a parallel coordinate system x'y'z' with its origin at the center of mass of the object.

the inertia matrix for the two booms in the xyz system is

$$[I] = \begin{bmatrix} 19200 & 86400 & 0 \\ 86400 & 1036800 & 0 \\ 0 & 0 & 1056000 \end{bmatrix} \text{kg-m}^2$$



The mass of the boom AB is mAB = 4800 kg.
The mass of the boom BC is mBC = 1600 kg.

Strategy: Find the location of the centre of mass in the xyz system, then apply the parallel axis theorems.

$$[I]_G = \begin{bmatrix} 156000 & 32400 & 0 \\ 32400 & 226800 & 0 \\ 0 & 0 & 242400 \end{bmatrix} \text{kg-m}^2.$$